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THERMAL STATE OF A POROUS PLATE UNDER COOLANT FILTRATION CONDITIONS

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The steady-state temperature field is considered in a finite porous plate during filtration of a coolant.

One of the important problems in the design of porous cooling systems is study of the temperature fields within porous bodies in the presence of filtration processes. In particular, [1, 2] were dedicated to this problem. Because of their complexity, problems of this type are solved in two stages: initially the dynamic problem is solved, i.e., the pressure (velocity) field in the porous body is found, after which the temperature field is constructed from the solution so obtained. Use of this approach is based on the assumption of the dominant effect of the velocity field on the temperature field in the porous body with only a weak effect in the opposite direction.

The goal of the present study is to obtain an analytical solution for the temperature field in a two-dimensional porous body when an incompressible liquid is used as the coolant.

We will consider the thermal state of a finite porous plate ABCDEK (Fig. 1a [3]), to which coolant is supplied through the face AC under a pressure P_1. A pressure P_2 exists on the surfaces AK and CD. In addition, we will assume the face KD to be impermeable and thermally nonconductive. Moreover, we assume that the temperature at the coolant input to the plate is equal to T_1, with temperature at the exit T_2, where T_2 is a known function of the coordinates.

The flow within the porous medium obeys the law

$$\frac{\alpha}{\mu(T)}\,\mathrm{grad}\,P = -\frac{f(v)}{v}\,\mathbf{v}, \qquad (1)$$

with consideration of which, heat and mass transport with a powerlike resistance law can be described by the equations

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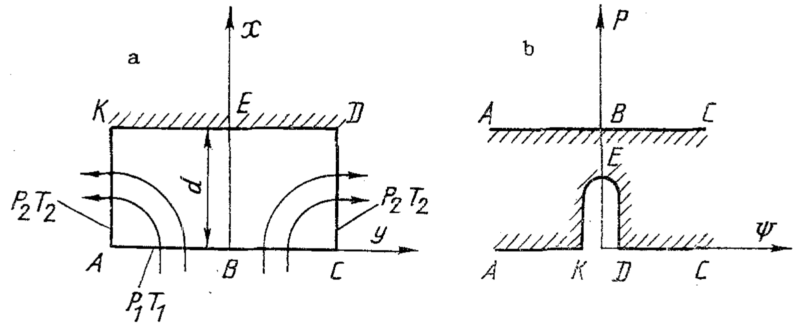

Fig. 1. Formulation of problem.

$$\lambda \Delta T - \rho c_p \mathbf{v} \operatorname{grad} T = 0, \tag{2}$$

$$\frac{\partial \Psi}{\partial \beta} = \frac{\sqrt{n+1}}{\chi} \exp(-2\varepsilon\tau) \frac{\partial P}{\partial \tau}, \quad \frac{\partial \Psi}{\partial \tau} = -\frac{\sqrt{n+1}}{\chi} \exp(-2\varepsilon\tau) \frac{\partial P}{\partial \beta}. \tag{3}$$

Solution of the dynamic problem can be reduced to solution of the Helmholtz equation for the exponential pressure function in Chaplygin coordinates τ, β. Subsequent application of a Fourier integral transform and the Wiener–Hopf method produces the following expressions for the unknown function [3]:

at $\tau > 0$

$$Q(\tau, \beta) = \frac{\frac{\pi}{2} - \beta}{\frac{\pi}{2}} \exp(-\varepsilon\tau) \frac{8\Phi(\varepsilon)}{\pi^2} \sum_{k=1}^{\infty} \frac{k \exp(-r_k \tau)}{r_k (r_k - \varepsilon) \Phi(r_k)} \sin 4k\beta, \tag{4}$$

at $\tau < 0$

$$Q(\tau, \beta) = \frac{1}{2} \exp(-\varepsilon\tau) + \frac{8\Phi(\varepsilon)}{\pi^2} \sum_{k=1}^{\infty} \frac{\Phi(s_k) \exp(s_k \tau)}{s_k (s_k - \varepsilon)} \cos 2(2k-1)\beta, \tag{5}$$

where

$$\varepsilon = \frac{n}{2\sqrt{n+1}}; \quad \Phi(z) = \prod_{k=1}^{\infty} \frac{2k}{2k-1} \frac{z + s_k}{z + r_k}; \quad r_k = \sqrt{16k^2 + \varepsilon^2};$$

$$s_k = \sqrt{4(2k-1)^2 + \varepsilon^2}.$$

Using Eqs. (4), (5) together with system (3) and the replacement

$$P = Q \exp(\varepsilon\tau) \tag{6}$$

we can find the pressure distribution in the filtration region

$$P(\tau, \beta) = \frac{1}{2} + \frac{8\Phi(\varepsilon)}{\pi^2} \sum_{k=1}^{\infty} \frac{\Phi(s_k) \exp[(s_k + \varepsilon)\tau]}{s_k (s_k - \varepsilon)} \cos 2(2k-1)\beta. \tag{7}$$

In particular, on BE(ED) Eq. (7) takes on the form

$$P(\tau, \beta) = \frac{1}{2} \pm \frac{8\Phi(\varepsilon)}{\pi^2} \sum_{k=1}^{\infty} \frac{\Phi(s_k) \exp[(s_k + \varepsilon)\tau]}{s_k (s_k - \varepsilon)}. \tag{8}$$

The pressure found in this manner appears as an independent variable in the differential equation for the temperature of the porous body

$$\frac{\partial}{\partial p}\left(\delta \frac{\partial T}{\partial p}\right) + \frac{\partial}{\partial \psi}\left(\frac{1}{\delta} \frac{\partial T}{\partial \psi}\right) + \frac{c_p}{\lambda} \frac{\partial T}{\partial p} = 0, \tag{9}$$

where $\delta = v''\mu/\alpha$.

In the case of variable coolant viscosity and nonlinear filtration, integration of Eq. (9) is difficult. However, if we assume $f(v)/v \simeq$ const, and take the coolant viscosity constant and equal to its mean value over the given temperature interval, then in the coordinates

$$P = p \quad \text{and} \quad \Psi = \frac{c_p \mu^2}{\alpha^2} \psi$$

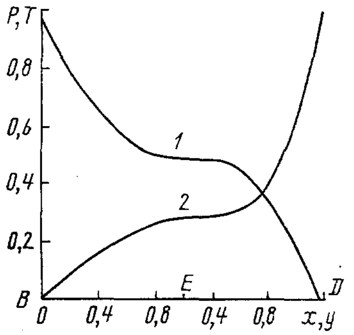

Fig. 2. Functions at n = 0.4: 1) P = f(x, y); 2) T = f(x, y) (along lines BE and ED).

Eq. (9) takes on the form

$$\frac{\partial^2 T}{\partial P^2} + \frac{\partial^2 T}{\partial \Psi^2} + \varkappa \frac{\partial T}{\partial P} = 0, \qquad (10)$$

where $\kappa = \alpha c_p/\mu\lambda$ = const. With the replacement

$$T = U \exp\left(-\frac{\varkappa}{2} P\right) \qquad (11)$$

we transform Eq. (10) to the Helmholtz equation

$$\frac{\partial^2 U}{\partial P^2} + \frac{\partial^2 U}{\partial \Psi^2} - \frac{\varkappa^2}{4} U = 0. \qquad (12)$$

If the impermeable boundary DK can be considered thermally nonconductive, then in the coordinates P, Ψ the filtration region forms an infinite band of width $0 \leq P \leq P_1$. The section along the P axis may be neglected, since the thermal load is symmetric. Moreover, since the impermeable boundary is nonconductive, along that boundary $\partial T/\partial \Psi = 0$ (Fig. 1b).

Let the temperature on the boundaries AK and DC be described by the equation

$$T_2 = T(\Psi, 0). \qquad (13)$$

We write the function U(P, Ψ) as a generalized Fourier integral:

$$U(P, \Psi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_+(P, \lambda) \exp(i\lambda\Psi)\, d\lambda + \frac{1}{2} \int_{-\infty+ib}^{+\infty+ib} F_-(P, \lambda) \exp(i\lambda\Psi)\, d\lambda, \qquad (14)$$

where

$$b > \frac{\varkappa^2}{4}; \quad F_+(P, \lambda) = \int_0^{\infty} U(P, \xi) \exp(-i\lambda\xi)\, d\xi;$$

$$F_-(P, \lambda) = \int_{-\infty}^{0} U(P, \xi) \exp(-i\lambda\xi)\, d\xi.$$

Here the function $F_+(P, \lambda)$ is regular in some lower half plane of the complex variable λ, while $F_-(P, \lambda)$ is regular in the upper half plane.

We then obtain the differential equation

$$\frac{d^2 F}{dP^2} - \eta^2 F = 0, \qquad (15)$$

where $\eta^2 = \lambda^2 + \kappa^2/4$. The boundary conditions are:

$$P = 0, \quad F = F(0, \lambda), \quad P = P_1, \quad F = 0. \qquad (16)$$

The solution of this equation has the form

$$F(P, \lambda) = F(0, \lambda) \frac{\operatorname{sh} \eta (P_1 - P)}{\operatorname{sh} \eta P_1}. \qquad (17)$$

The function U(P, Ψ) is defined by the expressions:

at Ψ > 0

$$U(P,\ \Psi)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}F(0,\ \lambda)\,\frac{\operatorname{sh}\eta(P_1-P)}{\operatorname{sh}\eta P_1}\exp(i\lambda\Psi)\,d\lambda, \qquad (18)$$

at $\Psi < 0$

$$U(P,\ \Psi)=\frac{1}{2\pi}\int\limits_{-\infty+ib}^{+\infty+ib}F(0,\ \lambda)\,\frac{\operatorname{sh}\eta(P_1-P)}{\operatorname{sh}\eta P_1}. \qquad (19)$$

In Eqs. (18), (19) for $\Psi > 0$ integration is performed over the upper λ half plane, and over the lower for $\Psi < 0$.

As an example, we will consider the steady-state temperature field in a porous plate when the temperature on the coolant exit surface is described by the expression

$$T=\exp(-N\,|\Psi|), \qquad (20)$$

where N = const. In this case the simple Fourier transform

$$F(0,\ \lambda)=F_+(0,\ \lambda)+F_-(0,\ \lambda)=\frac{2N}{\lambda^2+N^2} \qquad (21)$$

can be used. Then the reverse transform gives

$$U(P,\ \Psi)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\frac{2N}{\lambda^2N^2}\,\frac{\operatorname{sh}\eta(P_1-P)}{\operatorname{sh}\eta P_1}\exp(i\lambda\Psi)\,d\lambda. \qquad (22)$$

The integrand of Eq. (22) has simple poles at the points

$$\lambda=\pm iN,\quad \lambda=\pm iS_m,$$

where

$$S_m=\sqrt{\frac{m^2\pi^2}{P^2}+\frac{\varkappa^2}{4}}.$$

Then using residue theory and summing over m, we find

$$U(P,\ \Psi)=\frac{\sin\left[\sqrt{N^2-\frac{\varkappa^2}{4}}\,(P_1-P)\right]}{\sin\left(\sqrt{N^2-\frac{\varkappa^2}{4}}\,P_1\right)}\exp(-N\,|\Psi|)+\frac{2\pi N}{P_1^2}\sum_{m=1}^{\infty}\frac{m\exp(-S_m\,|\Psi|)}{(N^2-S_m^2)\,S_m}\sin\frac{m\pi P}{P_1}. \qquad (23)$$

As is evident from Eq. (23), the temperature distribution is symmetric about the axis $\Psi = 0$.

Figure 2 shows the pressure and temperature profiles along the plate axis and impermeable boundary (flow in $\Psi = 0$) at n = 0.4. It follows from the figure that the temperature distribution is nonlinear in character, with the steepness of the rise in the temperature curve decreasing continually along the line BE, but nevertheless increasing significantly with approach to the coolant extraction surface, i.e., to the zone of most intense heat exchange between the heated porous skeleton and the coolant filtering through the pores.

NOTATION

T, dimensionless temperature; c_p, specific heat of incompressible liquid; ρ, density; λ, effective thermal conductivity coefficient of porous body–coolant system; n + 1, degree of filtration (at n = 0 filtration is linear); $\Psi = \Psi^*/v_0d$, dimensionless flow function; $P = P^*/P_0$, dimensionless pressure; v_0, P_0, characteristic filtration velocity and pressure; f(v), function defining the filtration law in each concrete case; $\chi = (v_0{}^{n+1}d)/P_0\alpha$, dimensionless filtration parameter; d, characteristic dimension; $\mu(T)$, dynamic viscosity coefficient.

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